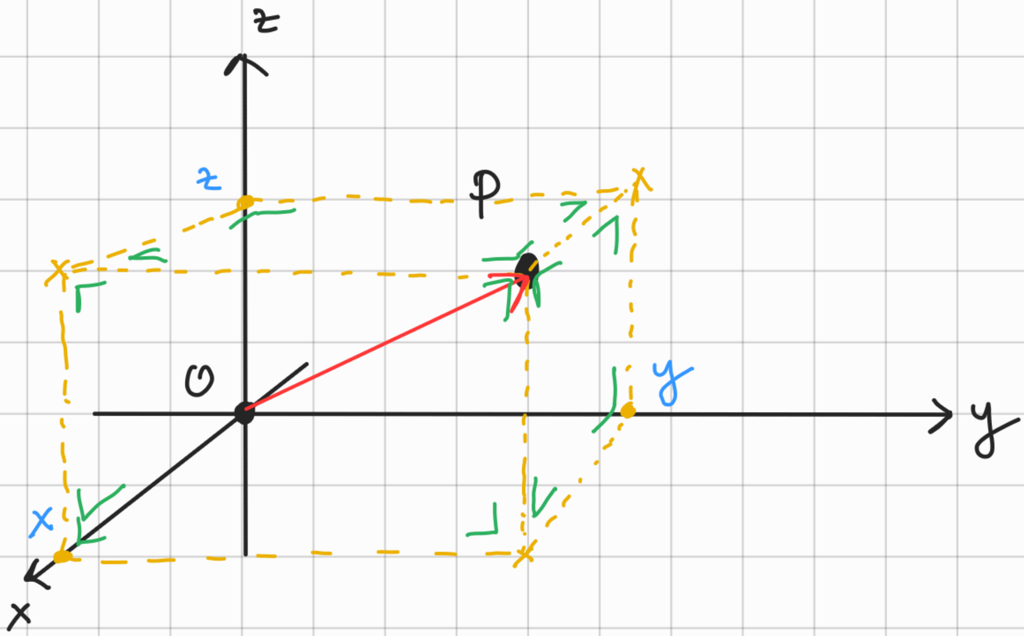
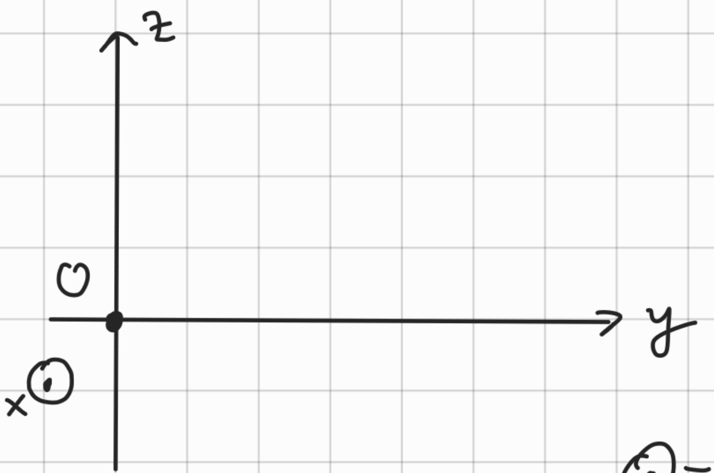
(11/10/2023)

G. Généralisations en 3d

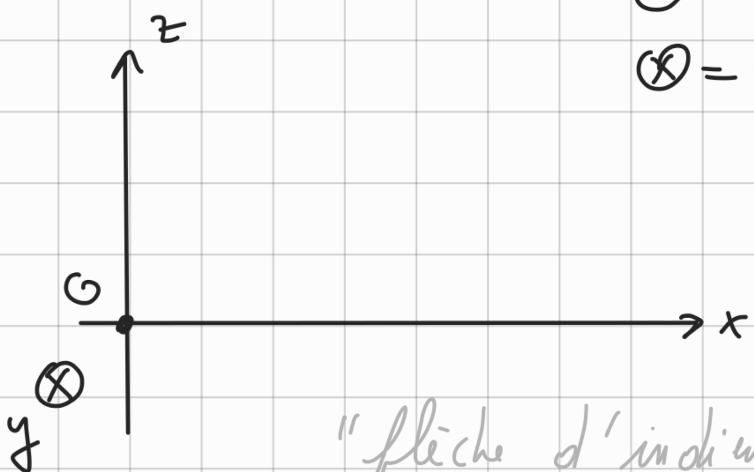


$$\overrightarrow{OP} = (x, y, z)$$



$\odot$ = axe vers nous

$\otimes$ = axe vers la feuille.



"flèche d'indien"

Question d'examen.

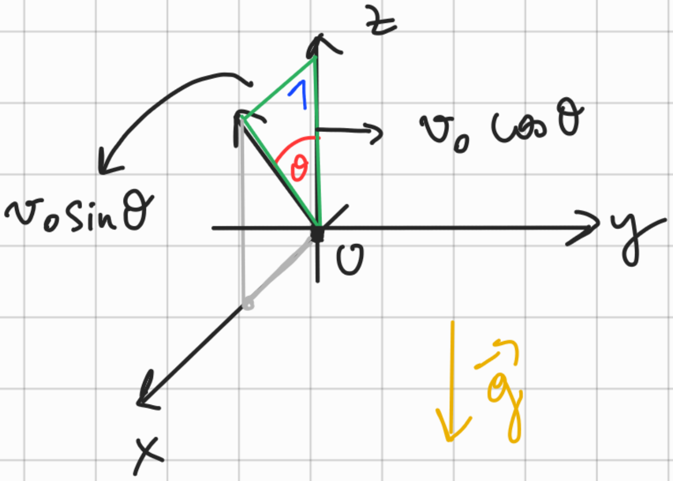
PHYS-G1103 2021|22 août

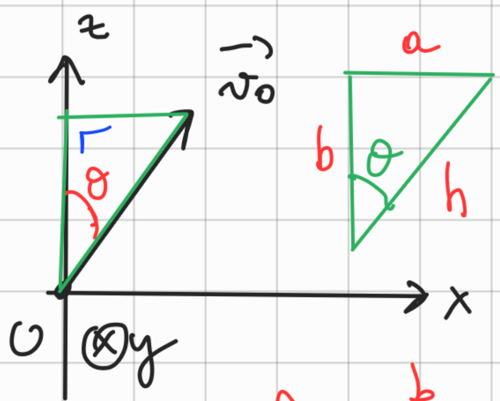
Q1.

1. $\vec{g} = (0, 0, -g)$ $\quad$ $\left[g = \|\vec{g}\| = 10 m/s^2. \right]$

$$\vec{g} = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix}$$

2.





$\cos\theta = \dfrac{b}{h}$

$h = \sqrt{a^2 + b^2}$

$$\vec{v}_0 = (v_0 \sin\theta, 0, v_0 \cos\theta)$$

$\sin\theta = \dfrac{a}{h}$

$$\cancel{\vec{v}_0 = (\sin\theta, 0, \cos\theta)}$$

$a = h \sin\theta$

$$\cancel{\vec{v}_0 = (v_0 \cos\theta, 0, v_0 \sin\theta)}$$

$v_x = v_0 \sin\theta$

3. Accélération constante $\Rightarrow$ MRUA:

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{g}t^2$$

$\vec{r}_0$: position initiale

Dans notre cas : $\vec{r}_0 = \vec{0}$

car par définition de O, c'est le point depuis lequel on lance le projectile.

$\vec{v}_0$: vitesse initiale.

→ comme dans l'énnoncé.

Composantes ?

$$\vec{r}(t) = \left(x_p(t), y_p(t), z_p(t)\right)$$

$$\begin{cases} x_p(t) = 0 + (v_0 \sin\theta)t + 0 \\ y_p(t) = 0 + 0 + 0 \\ z_p(t) = 0 + (v_0\cos\theta)t - \frac{1}{2}gt^2 \end{cases}$$

$+\frac{1}{2}\vec{g}t^2$

$= \frac{1}{2}t^2(0, 0, -g)$

$= (0, 0, -\frac{1}{2}gt^2)$

$\vec{r}_0 = (0, 0, 0)$ $\quad$ $\vec{v}_0 t = (v_0\sin\theta, 0, v_0\cos\theta)t$

$$\begin{cases} x_p(t) = v_0 \sin\theta \, t \\ y_p(t) = 0 \\ z_p(t) = v_0 \cos\theta \, t - \frac{1}{2} g t^2 \end{cases}$$

4. Pour le train, sa vitesse est constante $\Rightarrow$ MRU.

$$\vec{r}_T(t) = \vec{r}_{0T} + \vec{v}_T \, t$$

avec :

$$\vec{r}_{0T} = (d, \underbrace{r_{0Ty}}_{Y_0}, 0)$$

$$\vec{v}_T = \vec{V} = (0, -V, 0)$$

$$\Rightarrow \begin{cases} x_T(t) = d \\ y_T(t) = Y_0 - Vt \\ z_T(t) = 0 \end{cases}$$

5. Impact : $\vec{r}_p(t_*) = \vec{r}_T(t_*)$!

position du projectile

$$\begin{cases} x_p(t_*) = x_T(t_*) & (1) \\ y_p(t_*) = y_T(t_*) & (2) \\ z_p(t_*) = z_T(t_*) & (3) \end{cases}$$

$(1) \Leftrightarrow \quad v_0 \sin\theta \, t_* = d$

$(2) \Leftrightarrow \quad 0 = Y_0 - V t_*$

$(3) \Leftrightarrow \quad v_0 \cos\theta \, t_* - \frac{1}{2} g t_*^2 = 0$

$(2) \Leftrightarrow \quad t_* = \dfrac{Y_0}{V}$

$(3) \Leftrightarrow \quad v_0 \cos\theta - \frac{1}{2} g t_* = 0$

$$v_0 \cos\theta = \frac{1}{2} g t_* = \frac{1}{2} g \frac{Y_0}{V}.$$

$(1) \Leftrightarrow \quad v_0 \sin\theta = d/t_* = \dfrac{dV}{Y_0}.$

(a) (b)
$$\begin{cases} v_0 \sin\theta = \dfrac{dV}{Y_0} \longrightarrow \text{connu} \\ v_0 \cos\theta = \dfrac{g Y_0}{2V} \longrightarrow \text{connu} \end{cases}$$

$$\frac{v_0 \sin\theta}{v_0 \cos\theta} = \tan\theta = \frac{(dV/Y_0)}{(gY_0/2V)} = \frac{2dV^2}{gY_0^2}$$

$$\left[\frac{2dV^2}{gY_0^2}\right] = \frac{\cancel{L}\,\cancel{L^2}\,\cancel{T^{-2}}}{\cancel{L}\,\cancel{T^{-2}}\,\cancel{L^2}} = 1.$$

$$\theta = \arctan\left(\frac{2dV^2}{gY_0^2}\right).$$

$(a)^2 + (b)^2$

$$v_0^2 \sin^2\theta + v_0^2 \cos^2\theta = v_0^2$$

$$= \left(\frac{dV}{Y_0}\right)^2 + \left(\frac{gY_0}{2V}\right)^2$$

$$\Rightarrow v_0 = \sqrt{\left(\frac{dV}{Y_0}\right)^2 + \left(\frac{gY_0}{2V}\right)^2}$$

6. $d = 2\,m$ $\quad V = 15\,km/h$ $\quad Y_0 = 5\,m$.