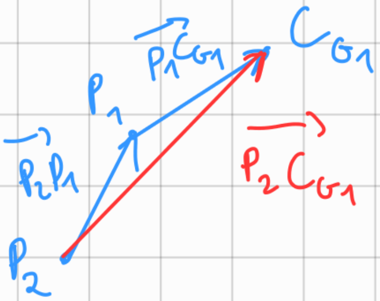
(13/12/2023)

(Suite correction q. examen
cf. cours n°20)

3). $\vec{\tau}_{P_2}(m\vec{g}) = \overrightarrow{P_2 C_{G_1}} \times (m\vec{g})$

$$\overrightarrow{P_2 C_{G_1}} = \overrightarrow{P_2 P_1} + \overrightarrow{P_1 C_{G_1}}$$



$$\overrightarrow{P_2 P_1} = (0, d, 0)$$

$$\overrightarrow{P_1 C_{G_1}} = \left(\frac{3L}{8}, -\frac{d}{2}, \frac{3h}{2}\right)$$

$$\overrightarrow{P_2 C_{G_1}} = (0, d, 0) + \left(\frac{3L}{8}, -\frac{d}{2}, \frac{3h}{2}\right)$$

$$= \left(\underset{A}{\frac{3L}{8}}, \underset{B}{\frac{d}{2}}, \underset{C}{\frac{3h}{2}}\right)$$

$$m\vec{g} = (0, 0, \underset{D}{-mg})$$

$$(A, B, C) \times (0, 0, D) = (BD, -AD, 0)$$

$$\overrightarrow{P_2 C_{G_1}} \times (m\vec{g}) = \left(\left(\frac{d}{2}\right)(-mg), \left(\frac{-3L}{8}\right)(-mg), 0\right)$$

$$\Rightarrow \boxed{\vec{\tau}_{P_2}(m\vec{g}) = mg\left(-\frac{d}{2}, \frac{3L}{8}, 0\right)}$$

4). $\vec{\tau}_{P_2}(M\vec{g}) = \overrightarrow{P_2 C_{G_2}} \times (M\vec{g})$

$$\overrightarrow{P_2 C_{G_2}} = \left(\underset{A}{\frac{3L}{4}}, \underset{B}{\frac{d}{5}}, \underset{C}{h}\right)$$

$$M\vec{g} = \left(0, 0, \underset{D}{-Mg}\right)$$

$$\Rightarrow \vec{\tau}_{P_2}(M\vec{g}) = \left(\left(\frac{d}{5}\right)(-Mg), \left(\frac{-3L}{4}\right)(-Mg), 0\right)$$

$$\boxed{\vec{\tau}_{P_2}(M\vec{g}) = Mg\left(-\frac{d}{5}, \frac{3L}{4}, 0\right)}$$

Remarque : calcul de ces moments de force avec la formule

$$\vec{A} \times \vec{B} = \left(A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x\right)$$

$$\vec{\tau}_{P_2}(\vec{N_1}) = \overrightarrow{P_2P_1} \times \vec{N_1}$$

$$\overrightarrow{P_2P_1} = (\underset{A_x}{0}, \underset{A_y}{d}, \underset{A_z}{0}) \qquad \vec{N_1} = (\underset{B_x}{0}, \underset{B_y}{0}, \underset{B_z}{N_1})$$

$$\Rightarrow \vec{\tau}_{P_2}(\vec{N_1}) = (\underbrace{d}_{A_y}\underbrace{N_1}_{B_z} - 0, 0 - 0, 0 - 0)$$

$$\boxed{\vec{\tau}_{P_2}(\vec{N_1}) = (dN_1, 0, 0)}$$

$$\vec{\tau}_{P_2}(\vec{N_3}) = \overrightarrow{P_2P_3} \times \vec{N_3}$$

$$\overrightarrow{P_2P_3} = (\underset{A_x}{L}, \underset{A_y}{0}, \underset{A_z}{0}) \qquad \vec{N_3} = (\underset{B_x}{0}, \underset{B_y}{0}, \underset{B_z}{N_3})$$

$$\vec{\tau}_{P_2}(\vec{N_3}) = (0 - 0, 0 - \underbrace{L N_3}_{A_x B_z}, 0 - 0)$$

$$\boxed{\vec{\tau}_{P_2}(\vec{N_3}) = (0, -LN_3, 0)}$$

$$(A, B, C) \times (0, 0, D) = (BD - 0,\ 0 - AD,\ 0 - 0)$$

$A_x \quad A_y \quad A_z \qquad B_x \quad B_y \quad B_z$

$$= (BD, -AD, 0, 0).$$

5). N_1, N_2, N_3 ?

On sait que $\vec{N}_1 + \vec{N}_2 + \vec{N}_3 = (0, 0, (m+M)g)$

$\vec{N}_1 = (0, 0, N_1) \qquad \vec{N}_2 = (0, 0, N_2)$

$\vec{N}_3 = (0, 0, N_3)$

Dir. x : $\quad 0 + 0 + 0 = 0$

Dir. y : $\quad 0 + 0 + 0 = 0$

Dir. z : $\quad N_1 + N_2 + N_3 = (m + M)g$

$\Rightarrow$ 1 équation pour 3 inconnues.

$\Rightarrow$ besoin de plus d'équations ...

Système immobile implique également que l'accélération angulaire est nulle.

$$\Rightarrow \vec{\tau}_{P_2} = \vec{0}$$

où $\vec{\tau}_{P_2}$ = moment de force total :

$$\vec{\tau}_{P_2} = \vec{\tau}_{P_2}(\vec{N_1}) + \vec{\tau}_{P_2}(\vec{N_2}) + \vec{\tau}_{P_2}(\vec{N_3}) + \vec{\tau}_{P_2}(m\vec{g}) + \vec{\tau}_{P_2}(M\vec{g})$$

$$= (dN_1, 0, 0) + \vec{0} + (0, -LN_3, 0) + mg\left(-\frac{d}{2}, \frac{3L}{8}, 0\right) + Mg\left(-\frac{d}{5}, \frac{3L}{4}, 0\right)$$

$$= \left(dN_1 - mg\frac{d}{2} - Mg\frac{d}{5},\ -LN_3 + mg\frac{3L}{8} + Mg\frac{3L}{4},\ 0\right)$$

$$= (0, 0, 0)$$

$$\Rightarrow \quad dN_1 - mg\frac{d}{2} - Mg\frac{d}{5} = 0 \qquad (Ox)$$

$$-LN_3 + mg\frac{3L}{8} + Mg\frac{3L}{4} = 0 \qquad (Oy)$$

$$0 = 0 \qquad (Oz)$$

On a donc 3 équations pour nos 3 inconnues N_1, N_2, N_3 :

$$\begin{cases} N_1 + N_2 + N_3 = (m+M)g & (1) \\ dN_1 - mg\frac{d}{2} - Mg\frac{d}{5} = 0 & (2) \\ -LN_3 + mg\frac{3L}{8} + Mg\frac{3L}{4} = 0 & (3) \end{cases}$$

$$(2) \Rightarrow N_1 = g\left(\frac{m}{2} + \frac{M}{5}\right)$$

$$(3) \Rightarrow N_3 = \frac{3}{4}g\left(\frac{m}{2} + M\right)$$

$$(1) \Rightarrow N_2 = (m+M)g - N_1 - N_3$$

$$= g\left[m + M - \frac{m}{2} - \frac{M}{5} - \frac{3m}{8} - \frac{3M}{4}\right]$$

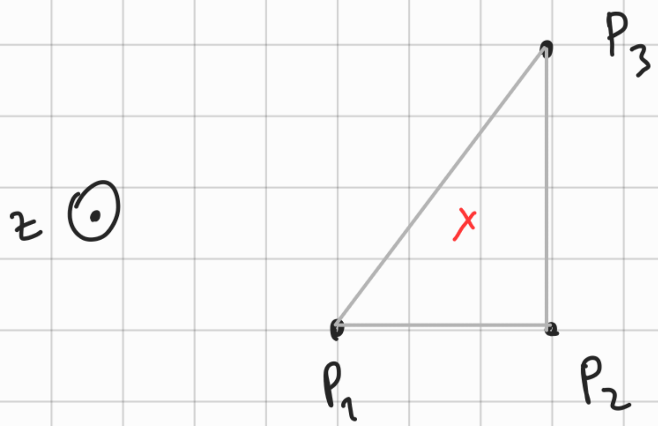
$$= g\left[m\left(1 - \frac{1}{2} - \frac{3}{8}\right) + M\left(1 - \frac{1}{5} - \frac{3}{4}\right)\right]$$

$$1 - \frac{1}{2} - \frac{3}{8} = \frac{8-4-3}{8} = \frac{1}{8}$$

$$1 - \frac{1}{5} - \frac{3}{4} = \frac{20-4-15}{20} = \frac{1}{20}$$

$$\Rightarrow N_2 = g\left(\frac{m}{8} + \frac{M}{20}\right)$$

$$\boxed{\begin{aligned} N_1 &= g\left(\frac{m}{2} + \frac{M}{5}\right) \\ N_2 &= g\left(\frac{m}{8} + \frac{M}{20}\right) \\ N_3 &= \frac{3}{4} g\left(\frac{m}{2} + M\right) \end{aligned}}$$



6). $\overrightarrow{P_2 C_{tot}} = \dfrac{1}{m+M}\left(m \overrightarrow{P_2 C_{G_1}} + M \overrightarrow{P_2 C_{G_2}}\right)$

$$\overrightarrow{P_2 C_{G_1}} = \left(\frac{3L}{8}, \frac{d}{2}, \frac{3h}{2}\right)$$

$$\overrightarrow{P_2 C_{G_2}} = \left(\frac{3L}{4}, \frac{d}{5}, h\right)$$

$$\overrightarrow{P_2 C_{tot}} = \frac{1}{m+M}\left(\left(m\frac{3L}{8}, m\frac{d}{2}, m\frac{3h}{2}\right) + \left(M\frac{3L}{4}, M\frac{d}{5}, Mh\right)\right)$$

$$= \frac{1}{m+M}\left(\frac{3L}{4}\left(\frac{m}{2}+M\right),\ d\left(\frac{m}{2}+\frac{M}{5}\right),\ h\left(\frac{3m}{2}+M\right)\right)$$